

Name:

Fall 2021 Stat 311 Exam 2

Instructions: Submit as a single PDF file titled P Last Name First Name Exam 1 (e.g. P Etkin Arkady Exam 1). Don't forget the P in front or you will turn into a goat! Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK! You are not presenting a valid Kafka Protocol at your peril.

1. A tabby cat gives birth to 5 kittens, 3 of which have the Siamese coat color pattern. The Himalayan gene responsible for this color pattern is recessive. This means that the 3 kittens with the Siamese phenotype must have received the Himalayan gene from both mother and father. It is estimated that carriers are 3 times as frequent as cats with the Siamese phenotype. Calculate the probability that the father exhibits the Siamese coat color pattern. Does the calculation match your intuition? [10 pts]

2. A marketing agency wants to call 1000 individuals to advertise their product. The agency has a pool of 1000 000 potential clients. Before each call a computer randomly selects the individual to be reached from the entire pool of 1000 000 names.

(a) Find an expression for the probability that at least one person is called several times. [4 pts]

(b) Give a good estimate of the probability in (a)

[6 pts]

3. A village researcher publishes an article titled Epidemic Of The Century. In this article, the researcher compares 5 of the most lethal outbreaks within the last 100 years and arrives at the conclusion that outbreak E is by far the deadliest. Specifically, the researcher claims that the chance of a lethal outcome from Epidemic E was higher than from any of the other 4 outbreaks. He deduces his conclusion from the following two tables:

Table 1: Crude Death

Age Group	A	B	C	D	E
0 - 20	10	10	9	8	6
21 - 40	38	60	88	115	72
41 - 50	55	36	14	30	72
51 - 60	7	18	55	39	68
61 - 70	15	15	30	14	48
71 - 80	10	30	17	57	100
81+	30	200	120	240	440
Total	165	369	377	503	806

Table 2: Demographic Data

Age Group	Year of A	Year of B	Year of C	Year of D	Year of E
0 - 20	2000	2500	3000	4000	6000
21 - 40	19000	20000	22000	23000	24000
41 - 50	11000	12000	14000	15000	18000
51 - 60	7000	9000	11000	13000	17000
61 - 70	3000	5000	6000	7000	12000
71 - 80	1000	1500	1700	1900	2500
81+	300	400	600	800	1100
Total	43 300	50 400	58 300	64 700	80 600

Do these tables support his conclusion? Why or why not? [Hint: You might find using Excel useful]

[10 pts]

4. 3 cards are randomly chosen without replacement from an ordinary deck of 52 cards. Given that all the chosen cards are aces, what's the probability that an ace of spades is among these 3 cards? [10 pts]

5. Let X be a Poisson random variable with $E[X] = \ln 3$. Compute $E[\cos(\pi X)]$. [10 pts]

6. A study concerning the relationship between blood pressure and heart abnormalities classifies the patient's blood pressure as High, Low, or Normal. The patient's heartbeat is described as either Regular or Not Regular. The following statistics were obtained:

- (i) 14% of patients in the study have high blood pressure.
- (ii) 22% have low blood pressure.
- (iii) 15% have an irregular (Not Regular) heart beat.
- (iv) Of those with an irregular heartbeat, one-third have high blood pressure.
- (v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

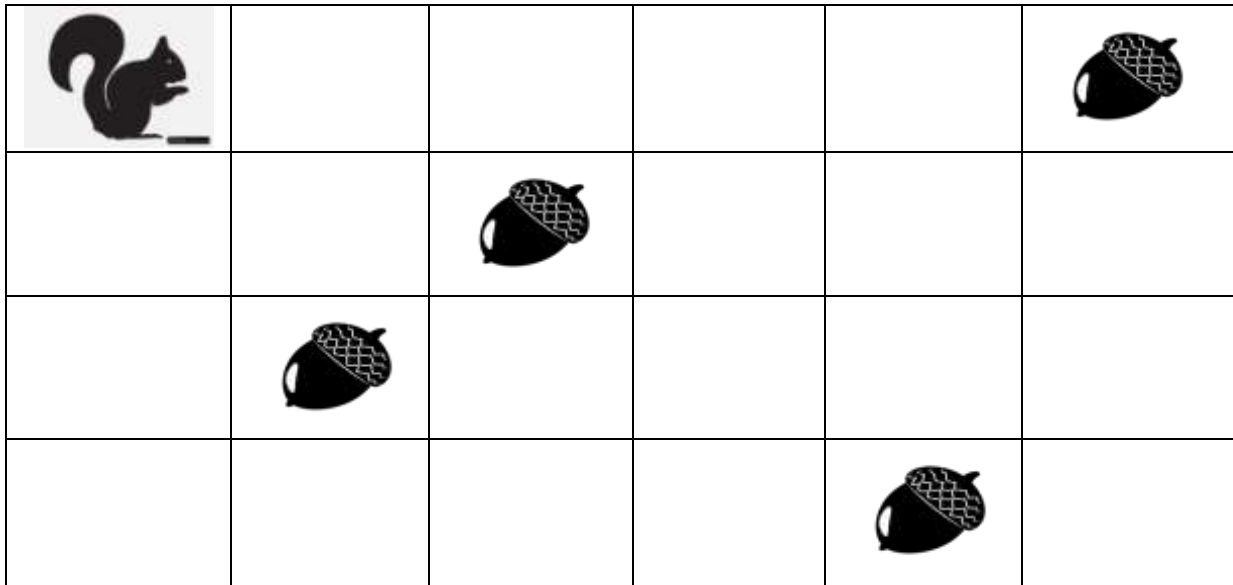
What proportion of patients have a regular heartbeat and low blood pressure? [10 pts]

7. A ball is equally likely to be painted yellow or blue. Two such balls are placed in an urn. Given that one of the balls is yellow, what's the probability that both balls are yellow? [10 pts]

8. An experiment consists in rolling a fair die n times and calculating the ratio $\frac{n(6)}{n}$ that is the number of times the result was a 6 divided by total trials. How large should n be to insure that with 99% certainty $\left| \frac{n(6)}{n} - \frac{1}{6} \right| \leq 0.01$? [10 pts]

9. n words are randomly chosen from the set {mortal, coil, this, shuffle, off, to, be, not, or}. How many times do you expect the sentence "to be or not to be" to occur? [10 pts]

10. A squirrel has hidden an acorn in 4 out of 24 sectors of a 6×4 rectangular grid. Now, with the coming of winter she is trying to find them. The squirrel does not remember which sector she searched; having visited one sector, the squirrel randomly selects any one of the 24 sectors for her next search. The squirrel stops once all the acorns are recovered. Assuming that the squirrel finds each acorn upon the very first visitation of the area where it is stored, compute the expected number of sectors that she will need to visit. [10 pts]



Extra Credit

11. You plan to dine in an unknown restaurant m times. There are $n \geq m$ dishes on the menu. If you had tasted every single dish, you could rank them according to your preference from 1 to n , with dish ranked as n being your favorite. To maximize the enjoyment from your dining experience, you will order $1 \leq k \leq m$ random dishes on the first k visits and repeatedly order the best tasting dish from among the k explored for the remaining $m - k$ visits. What is the value of k that maximizes the expected sum of the rankings for the dishes sampled? [10 pts]

12. During Hanukah it is customary to give children money (*demei Chanuka* דמי חנוכה) and spin the Dreidel - a 4 sided toy marked with the Hebrew letters ש נ ג ה (There = שם, was = היה, great = גדול, miracle = נס). See picture. The child is offered to play one of two games:

Game 1: Spin the Dreidel until two consecutive outcomes read "Great Miracle" and get paid \$1 for each round the Dreidel was spun.

Game 2: Spin the Dreidel until two consecutive outcomes read "Miracle Miracle" and get paid \$1 for each round the Dreidel was spun.

Which game should the child choose?

[10 pts]

